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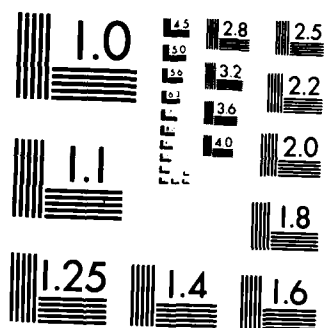   APPLIED SCIENCE S B BATDORF ET AL. 17 JAN 85
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MICROCOPY RESOLUTION TEST CHART

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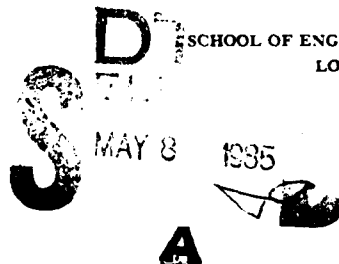
MAY 8 1985

Dr. Yapa Rajapakse   17 Jan. 1985
Office of Naval Research (Code 432)
800 N. Quincy St.,
Arlington VA 22217

Dear Dr. Rajapakse

This letter is intended to serve as the final report on UCLA contract No. N00014-76-C-0445, entitled "Statistical Fracture Theories for Brittle Materials", on which Dr. Batdorf served as Principal Investigator, and Dr. Sines as CoPrincipal Investigator.

Dr. Batdorf's activities in direct support of Navy interests began about 1974 when he was serving as Chief Scientist of The Aerospace Corp. at El Segundo. Under the sponsorship of the SAMSO program on Advanced Ballistic Reentry Systems (ABRES) he developed an inelastic polyaxial stress strain law that accurately matched the available stress strain data for ATJS graphite [1]. Since this was a material widely used in the nosetips of ballistic missiles, the stress strain law was adopted by the Trident structures people at Lockheed Missile and Space Co. and incorporated into their FANCAP (Fully Automated Nosecap Analysis Program) computer code, which was used extensively to study the survivability of various candidate designs under various reentry conditions. This stress strain relation, with the constants suitably adjusted, has been found equally applicable to later bulk graphites, and FANCAP has been the main workhorse for survivability studies of this type ever since.

Dr. Batdorf was also studying the statistical fracture behavior of brittle materials in general and graphite in particular. A typical problem faced by the designer is to take uniaxial fracture data obtained on small standard test specimens and use it to estimate the failure probability of a much larger structure of different shape under different stress conditions. Since missiles must meet a specified minimum survival rate in service, this subject is of vital importance in missile design, and Lockheed wanted more work

in this area than ABRES could afford to support. So Navy
Capt. Dickey of the Trident Project wrote a letter to Dr.
Perrone (see attachment) requesting ONR sponsorship. This
letter started the present program. The program was later
transferred to UCLA, where Dr. Batdorf had a simultaneous
appointment as an Adjunct Professor. The project ended Sept.
31, 1984.

HIGHLIGHTS- HOMOGENEOUS MATERIALS

At the time Dr. Batdorf began his research on statistical
fracture theory it was in a quite unsatisfactory condition.
The subject had advanced very little since Weibull published
his papers on the subject in 1939 [2,3]. Weibull's theory
was generally accepted for uniaxial stress states, but there
were strong differences of opinion on how to treat polyaxial
stress states. Weibull's recommendations regarding polyaxial
stress states were based on intuitive arguments that were
rejected by most practicing engineers. Instead the majority
of them treated this case by assuming that the survival
probability was the product of the survival probabilities of
the three principal stresses applied individually. Neither
of these approaches gave any consideration to the actual
nature of the flaws responsible for failure.

Dr. Batdorf assumed that the flaws were microcracks, and
initially employed the simplifying assumption that the
failure of a crack depended only on the component of stress
normal to the crack plane. This led to a new theory that for
the first time rested on a rational physical foundation. The
theory was found to be in good agreement with experiments on
Poco graphite [4], one of the candidate materials for
reentry nosetips at that time.

Colleagues at UCLA undertook to check the applicability of
the new theory to ceramics experimentally, under NSF
sponsorship. Dr. Sines compared theory and experiment in the
case of uniaxial and equibiaxial bending of high strength
alumina [5,6]. Whereas Griffith's theory for polyaxial
stresses predicted the strength would be the same for both
stress states, Dr. Sines found that alumina is weaker in the
equibiaxial state, in agreement with the predictions of the
new theory. Dr. Knapp tested low strength porous alumina
plates in combined longitudinal bending and transverse
compression [7], and obtained results closer to the Griffith
prediction as modified by Babel and Sines for ellipsoidal
cavities [8].

Now it is known that the failure of cracks depends not only
on the direct stress but also on the shear stress acting on
the crack plane. But the fracture mechanics community was
not (and even now is not) in agreement as to the proper form
of the shear dependence. The theory was therefore
reformulated for arbitrary fracture criterion [9]. It was

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also discovered that although the mathematical form of the new theory was very different from that of Weibull, the Weibull theory was a special case of the new theory-- the two agreed if the fracture criterion employed in the new theory was that only the direct component of stress contributed to fracture.

A byeproduct of the collaboration between Drs. Batdorf and Sines was the discovery of a new theorem in linear regression analysis [10]. This theorem shows how to obtain a least squares fit to two or more straight lines constrained to have a common slope. This is useful not only in many applications of Weibull theory-- reduction of experimental data involving specimens of different size or under different stress states, fatigue data, etc.-- but in many other types of problem.

Several papers followed in which the statistical consequences of various fracture criteria that have been proposed were examined and compared with experimental data. It was found that even though its physical justification seems shaky, Sih's energy density theory seems to be in better agreement with the limited available statistical fracture data than any other well-known fracture criterion [11,12].

In 1978 Dr. Alan Kushner recommended applying statistical fracture theory to the problem of finding the failure probability of a radome or irdome under laser attack. It turned out that the experimental scatter in the flat plate data obtained by Freiman and Mecholsky at NRL could be accounted for very nicely by statistical fracture theory, and that curves could be drawn for the probability of failure as a function of the total absorbed radar flux. Right after this both investigators left and NRL discontinued the effort. John Koenig of AFML requested us to help interpret his data on the same subject when they obtained enough of it, but it didn't happen because shortly after this their effort was discontinued. We gave a seminar on the subject to interested parties at General Dynamics, Pomona, to help them in studies aimed at upgrading their Navy missile, but have not published on the subject.

Several invited tutorial papers on statistical fracture theory have been prepared and presented at various places. One was the opening paper at the Second International Conference on Ceramic Fracture at Penn State [14]; another was the ONR International Conference on Fracture Mechanics [15]; still another was the section on Statistical Fracture Theory for Pergamon Press's new Encyclopedia of Material Science and Technology. Also at the request of DOE, Drs. Sines and Batdorf gave a two-day lecture on brittle material design technology at the Idaho Falls National Engineering Lab, managed by EG&G. In addition it is understood that a

group at the University of Washington has picked up our ideas and is working on a handbook of brittle design technology under NASA sponsorship.

One of the consequences of the assumption that the flaws causing fracture are cracks is that a flaw of given size will get weaker as it approaches a free surface. A necessarily crude theoretical analysis of this situation was made using the assumption that all cracks are normal to the surface and to the direction of stress. It was found that if the surface was carefully prepared so it is free of scratches and residual stresses, and if the flaws in the interior are Weibull-distributed, the flaws on or near the surface will also be Weibull-distributed, but with a Weibull shape parameter two less than that for the interior flaws [16].

One of the major unsolved problems of statistical fracture theory is to find the stress corresponding to a very low probablility of failure, say one in million, without testing a million specimens. The usual approach is to assume that the Weibull-type analytical crack strength distribution that best fits the data over the limited stress range of the available tests continues to be valid at the much lower stress level being sought. But this is without physical justification.

The best way of approaching this problem in all probability is to tie the crack size distribution, and through it the crack strength distribution, to the microstructure, concerning which much is known and much more can be discovered nondestructively in most cases of practical interest. McClintock proposed for this purpose the assumption that cracks are simply random aggregations of poorly bonded or unbonded pairs of neighboring cracks, and he applied this approach to a two-dimensional idealization of a solid body [17]. Early in the present program theory was extended to a three-dimensional model of a solid body [18] and the predictions compared with Weibull theory and experiment. Among the more interesting results: (1) The total number of cracks was found to be finite (the infinite number implied by Weibull's distribution functions is theoretically untenable) (2) There is a size effect, but less than that predicted by Weibull theory (experiments generally lead to the same conclusion). Although this new microstrure-based theory was the subject of an invited paper at the SMIRT Conference in London in 1975, it was not pursued further because most experimentalists believe that flaws causing failure are generally not of the nature assumed by McClintock. In some cases they are probably pores or inclusions, or collections of them; in others they are shrinkage cracks; in still others (as in graphites) they are lenticular cracks. This makes it appear likely that different substances behave somewhat differently in their

statistical fracture characteristics. This lead has not been actively pursued because of a series of technical breakthroughs in our understanding of composites, which was a field generally considered to be of higher priority. This will be described next.

HIGHLIGHTS- COMPOSITES

In conventional weakest link theory for brittle materials failure is considered to be due to preexisting flaws which are unaffected by the rising stress until one reaches instability and grows catastrophically. In unnotched fibrous composites, cracks are caused by the applied stress, and grow (and therefore weaken) as the stress increases. Failure occurs when the weakest crack becomes unstable. The resulting statistics of failure are quite different from those of strictly brittle materials.

The theory of composite failure was initiated by Rosen [19] and improved by Zweben [20,21]. In striving for accuracy, however, they got bogged down in mathematical detail and were unable to carry out calculations of crack growth parametrically beyond double fiber fractures. Tamuzs [22] and McKee and Sines [23] have treated particular cases numerically and carried the calculations out to final failure, which occurred in most cases when the crack causing failure was an 8-fold to 20-fold fiber fracture. All of these calculations involved approximations.

Phoenix et al [22-25] discovered a virtually exact technique for finding the failure stress of a composite involving only a linear array of fibers. Unfortunately the solution depends on the employment of a rather crude approximation to the stress concentration at the crack tip, which had been found for a few cases by Hedgepeth and Van Dyke [28,29]. Nevertheless the solution revealed for the first time the general nature of the relationship between conventional weakest link theory and the behavior of uniaxially reinforced composites.

Batdorf published the first general parametric solution for multiple ply uniaxial composites in which correct stress concentration factors could be employed [30]. One of its unique features was that it abandoned the chain of minibundles model which was introduced by Rosen and employed by most other investigators since, and concentrated instead on the creation and growth of randomly located cracks. The theory was shown to be in fairly good agreement with the limited available test data in [31]. A conspicuous difference between theory and the real world is that theory generally assumes that the fibers form a perfectly regular array (e.g., linear, square, hexagonal, etc.) whereas real composites tend to be quite irregular. The theory was therefore extended to cover the case of randomly irregular
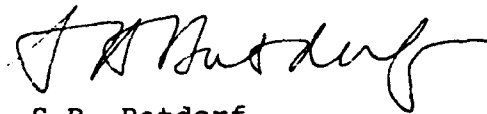
arrays [32,32]. This was found to improve agreement in some cases and not in others; probably it is a second order effect in the case of well constructed composites.

All theories for the growth of cracks must depend on a knowledge of the stress distribution in the vicinity of the crack tips. Most of the very limited knowledge available on this subject was obtained with the aid of shear lag theory, which is an approximate theory is of uncertain accuracy and which contains a shear coupling parameter that is difficult to evaluate. An electric analog to the stress distribution was discovered [34] which is in some respects similar to shear lag but is more accurate. The analog has been exploited to evaluate the shear coupling parameter appearing in shear lag theory for the cases of linear, square, and rectangular fiber arrays [34-36]. The potential of the technique for finding stress distributions has not been adequately evaluated for a number of reasons, such as (a) lack of an exact (or nearly exact) theoretical solution with which to compare this new experimental approach; (b) inadequate electrical measurement facilities and graduate students with the proper skills; and (c) an accident suffered in the machine shop by the graduate student assigned to work on this problem, that prevented him from doing any experimental work during the last six months of the contract.

Among the most important consequences of the new theory are (1) The size effect becomes progressively less as the volume increases (2) The variability in fracture strength tends toward zero as volume increases (3) The critical crack size increases as volume increases. The first prediction is in general agreement with the available data. The second is not; variability decreases but levels out at a finite value, probably because other sources of variance not contemplated in the theory , such as residual stresses, are present. The third prediction (as well as the predicted crack size distribution as a function of applied stress) has not been experimentally checked out because nobody has as yet discovered a good method for determining crack sizes in composites.

As will be evident from the preceding discussion, the main features of damage accumulation and failure in uniaxial composites subjected to tension are now fairly well understood. By way of contrast, in crossply laminates while much is known about damage accumulation, the final failure, which controlled by the 0-degree plies, is not understood. It is reasonable to hope that the extensive experience with failure mechanics of uniaxial composites (entirely 0 plies) resulting from the studies in this contract will enable us to provide this very important missing link in the failure mechanics of crossply laminates.

Sincerely,

S.B. Batdorf
Principal Investigator

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Dr. Nicholas Perrone   7 August 1974
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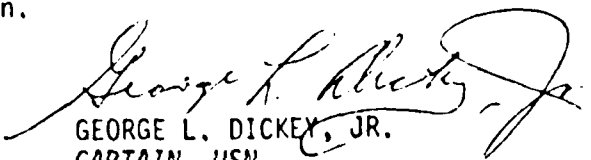806 N. Quincy Street
Arlington VA   22203

Dear Dr. Perrone

Several months ago, I was asked by the Reentry Systems people in
the Strategic Systems Project Office to obtain the services of
Dr. Sam Batdorf to assist in a TRIDENT system development problem.
In particular, they wished to put to practical use some of his
theoretical work on material fractures. The ABRES director agreed
that this support to the Navy was within the ABRES charter, and
Dr. Batdorf began his efforts without delay.

More recently, Dr. Batdorf has spoken to you about additional sup-
port, but from ONR. Knowing this, I spoke with the Reentry Systems
people at Lockheed, the actual users of Dr. Batdorf's work, as a
follow-up. I was assured by the appropriate people that they were,
indeed, able to put his results to good use in helping to solve
immediate TRIDENT materials problems. Not only do they wish to
retain his services, they encouraged me to lend support to his car-
rying out further theoretical work, especially in formulating stress-
strain relationships. It is quite gratifying to find a long-term
theoretical investigation which helps to solve a real-world problem,
and in a timely fashion.

Both SSPO and Lockheed were willing to pay for half of Dr. Batdorf's
time, representing the Advanced and Engineering Development aspects
of his work. We determined that it is much more efficient for
ABRES to do so, and ABRES will take advantage of the results. The
fact that he is making a contribution to TRIDENT today is a strong
argument for continuing his theoretical work, which is more related
to ONR sponsorship. Therefore, ABRES will fund half, and I urge you
to give consideration to handling the remainder. In addition, I sug-
gest that you, and others in ONR who are involved in materials and
structures, visit SAMSO in order to become aware of ABRES programs
which you should be interested in.

cc: Dr. Batdorf   GEORGE L. DICKEY, JR.
     RS   CAPTAIN, USN
     Dr. Hartunian   Asst Deputy for Naval Applications
     SP-272

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